CONTROL-THEORY HEURISTICS FOR IMPROVING THE BEHAVIOR OF ECONOMIC MODELS

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Abstract

The usefulness of control theory techniques in economic simulations is examined. A model of the capital investment accelerator is used for illustration. It is shown that behavior approaching optimality can be achieved in a highly nonlinear system with simple heuristics based on linearization and constant state feedback with an appropriate choice of desired closed-loop poles. Applications to real life situations are discussed and suggestions are made for approaching potential problems using similar control theory techniques.

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Introduction

One can approach a system dynamics problem from two extreme points of view: mathematical and intuitive. Mathematical analysis provides global knowledge of the solutions but is only feasible in simple models. Thus, it is very hard to analyze large, realistic models mathematically. On the other hand, with the intuitive approach, one can investigate more comprehensive and realistic models, but it is all too easy to become lost in simulation and trial and error in designing policies to improve performance. We seek to answer several questions:

1. How useful are the techniques of modern control theory in simulation models of social systems?
2. Can "approximately optimal" policies be designed which provide significant performance improvement relative to the original policies of the decisionmakers? By "approximately optimal" is meant a set of policies based on heuristic methods rather than the closed-form solution of the problem, since such analytic solutions are generally impossible to determine in the high order, nonlinear systems typically of concern in system dynamics modelling.
3. How different are the improved and original policies? That is, are the improved rules consistent with the likely availability of information and bounded rationality of real managers?

The goal of this paper is to motivate the heuristic use of some mathematical tools to aid the systems analyst. The approach can be outlined as follows:

1. Start from the full simulation model portraying the problem of interest.
2. Analyze the model formally, using decoupling, model reduction, linearization, etc., where appropriate to decrease the burden of the mathematical analysis.
3. Design an appropriate controller (policies for improved performance).
4. Project the results back to the original model and test the robustness of the new policies in the full system.

We assume that 1 is given and concentrate on 2, 3, and 4. The paper complements previous applications of control theory to policy design in social and corporate systems (e.g. Mohapatra and Sharma 1985, Coyle 1985, Kivijarvi and Tuominen 1986) by applying formal methods to a nonlinear model with complex dynamics. Further, the existence of experimental results showing how people actually manage the system provides a means for directly assessing the performance improvements which may be obtained through the use of control techniques.

We next describe some of the basic mathematical tools. The above approach is applied to Sterman's (1985) model of the Kondratiev cycle or long wave. The particular example illustrates general techniques applicable to a variety of situations such as business cycles (Mass 1975, Lyneis 1980), commodity price stabilization (Meadows 1969), market growth (Forrester 1968) and others. Although this paper is mostly heuristic, we hope to motivate the need for rigorous analysis based on the outlined method. The last section summarizes our results and makes recommendations for further analysis.

Mathematical Tools

Linearization is the most common tool for formal analysis of nonlinear models. A model of the form

$$\dot{x}(t) = f(x(t)) + g(u(t)) \tag{1}$$

where x is the vector of states, u is the vector of inputs in the model and $f(\cdot)$, $g(\cdot)$ state and input functions, respectively, could be linearized around a nominal trajectory $x_n(t)$ using the Jacobian matrices

$$F = [f_{i,j}], \quad f_{i,j} = \partial f_i(x)/\partial x_j \tag{2.a}$$

$$G = [g_{i,j}], \quad g_{i,j} = \partial g_i(u)/\partial u_j \tag{2.b}$$

Then the linearized model

$$\delta\dot{x}(t) = F\delta x(t) + G\delta u(t) \tag{3}$$

describes deviations from the nominal trajectory. The actual trajectory $x(t) = x_n(t)+\delta x(t)$ is driven by the input $u(t) = u_n(t)+\delta u(t)$ where $f(x_n(t))+g(u_n(t)) \simeq 0$. This approximation is good for "slowly varying" nominal trajectories. The model can be analyzed using modern control theory (Guckenheimer and Holmes 1983; for "slowly varying" see Vidyasagar 1978).

It should always be remembered that linearization only provides information about the local region of phase space, and is not a reliable guide to global dynamics. A special class of nonlinearities is the class of piecewise linear functions, such as saturation, bang-bang, roundoff etc. Their derivatives are discontinuous, and therefore the Jacobian matrix is undefined. In this case, the analysis can be performed for different regions where the derivatives of these nonlinearities are defined.

Once the linearized model (3) is constructed, various methods of control can be applied. The method used in this paper is pole placement using full state feedback (Kailath 1980). In essence, a weighted sum of each state is fed back to the system as the input. The method is to calculate these weights, H, in order to achieve a given set of closed loop poles (i.e. the eigenvalues of F+GH). If the system (3) is controllable, then a matrix H can be chosen such that the eigenvalues of F+GH are as desired. The input $\delta u(t)$ is picked as:

$$\delta u(t) = H\delta x(t) \tag{4}$$

The closed loop system then becomes:

$$\delta \dot{x}(t) = (F+GH)\delta x(t) \tag{5}$$

and has the desired closed loop poles. The nominal input is picked to sustain the nominal trajectory. For example, the nominal trajectory might be given by the equilibrium of the system, which may be changing with exogeneous conditions.

To achieve an approximately optimal response for a system with 2 poles, one rule of thumb is to select a pair of poles for the closed loop system such that these poles are complex conjugates with equal magnitude of real and imaginary parts and negative real part. The response to a step input exhibits a slight overshoot and is optimal in the sense of minimum overshoot and settling time (Roberge 1975). The distance from the poles to the origin is a trade-off between overshoot and settling time.

An important aspect of any control design is robustness. We consider robustness by perturbation of the control parameters and by introducing uncertainty in the states.

A Model of the Kondratiev Cycle

Various models of the Kondratiev cycle or long wave have been developed to analyze recent economic difficulties (Rasmussen, Mosekilde, and Sterman 1985; Sterman 1985; Sterman 1986; Vasko 1987). This paper adopts the model of Sterman 1985. The Kondratiev model is chosen since it is a nonlinear system which exhibits complex dynamics, yet its structure is simple enough to allow for a considerable amount of mathematical analysis to be done by hand to illustrate the proposed techniques. It is also well known in the systems dynamics literature. The model has also been converted into a simulation game, Strategem-2 (Sterman and Meadows 1985). Strategem-2 provides a simulated economy in which the human decisionmakers replace the decision rule of the original model. Thus the game constitutes a laboratory for experimental testing of the model. Strategem-2 motivates the design of an "optimal" controller to "play" the game as we can compare the behavior of actual managers to the "optimal" response, thus providing a rough measure of the value of improved performance through formal analysis.

The long-wave model portrays the process governing capital investment in the aggregate economy. The capital producing sector strives to satisfy the demand for capital of the goods producing sector as well as its own capital needs. The model represents the capital "self-ordering" feedbacks created by the fact that capital is an input to its own production. This dependency implies that the total demand for capital can only be filled when there is sufficient capacity, but also that the capacity of the capital-producing sector can only be increased by first ordering additional capacity and adding to the demand. Intuitively, such a positive feedback must destabilize the adjustment of capital producers to shocks, and in fact, the model typically exhibits large-amplitude limit cycles. It can also generate chaotic behavior, where no periodicity can be attributed to the model (Sterman 1988b). In the experimental version, subjects play the role of managers for the capital producing sector and strive to balance the supply and demand for capital. Subjects seek to minimize their costs or "score" during each trial. The score is the average absolute deviation between supply (production capacity, PC) and demand (desired production, DP) over the T periods of the game: $S = \frac{1}{T}\Sigma|DP_t - PC_t|$. When presented with an unanticipated step input in demand, a sample of about 50 subjects produced an average score of more than 500 (Sterman 1987, 1988a). The optimal score for the same situation is 19.[1] The poor

[1]The optimal score was determined by grid search of the score space as a function of successive decisions. The optimal strategy presumes that the step in demand is unanticipated, consistent with

performance of the subjects shows that even an approximately optimal controller may offer substantial improvements in performance.

The states in the model are capital (K), the backlog of orders placed by the capital sector (BKS), and the backlog of orders placed by the goods sector (BGS). The inputs are new orders from the capital sector NKS (specified by the player), and new orders of the goods sector NGS (exogenous). The model can be summarized as follows (Figure 1):

$$\begin{bmatrix} \dot{K} \\ \dot{BKS} \\ \dot{BGS} \end{bmatrix} = \begin{bmatrix} \frac{BKS}{NCAT}\,FDS - \frac{K}{ALC} \\ -\frac{BKS}{NCAT}\,FDS \\ -\frac{BGS}{NCAT}\,FDS \end{bmatrix} + \begin{bmatrix} 0 \\ NKS \\ NGS \end{bmatrix} \tag{6.a}$$

$$\begin{aligned} FDS &= \text{Fraction of Demand Satisfied} \\ &= \frac{\text{Min(Desired Production, Capacity)}}{\text{Desired Production}} \end{aligned} \tag{6.b}$$

$$\text{Desired Production} = P^{*} = \frac{BKS + BGS}{NCAT} \tag{6.c}$$

$$\text{Capacity} = \frac{K}{COR} \tag{6.d}$$

where the normal capital acquisition time (NCAT), average lifetime of capital (ALC), and capital output ratio (COR) are constants. For the experiment in this paper, the values NCAT = 2, ALC = 20 and COR = 2 are used; these are the values used in the Strategem-2 game. Also, the same time step (2 years), simulation length (70 years), and score function, as the game, were used in order to compare our results to the game. The acquisition lag and capital/output ratio

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the information available to the subjects.

were deliberately set equal to the time step to facilitate play in the STRATEGEM-2 game. In the original model the parameters were based on econometric evidence and the time step reduced so that integration error was not significant. The difference and differential equation versions of the model produce the same types of behavior and respond to parameter changes in the same fashion.

Linearization:

The only nonlinearity requiring piecewise analysis arises from the fraction of demand satisfied, FDS. The capital-producing sector can only fully satisfy the demand when that demand is less than or equal to capacity. FDS equals 100% when capacity exceeds desired production. As desired production rises above capacity, FDS falls, constraining the growth of capital itself. The model operates in two distinct regimes: FDS = 1 (capacity exceeds demand), and FDS < 1 (insufficient capacity).

Considering each regime in turn, linearization of Equation (6) yields:

1. For FDS = 1,

$$\begin{bmatrix} \delta\dot{K} \\ \delta\dot{BKS} \\ \delta\dot{BGS} \end{bmatrix} = F_2 \begin{bmatrix} \delta K \\ \delta BKS \\ \delta BGS \end{bmatrix} + G_2 \begin{bmatrix} \delta NKS \\ \delta NGS \end{bmatrix} \tag{7}$$

$$\text{where } F_2 = \begin{bmatrix} -1/ALC & 1/NCAT & 0 \\ 0 & -1/NCAT & 0 \\ 0 & 0 & -1/NCAT \end{bmatrix} \text{ and } G_2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

2. For FDS < 1

$$\begin{bmatrix} \dot{\delta K} \\ \dot{\delta BKS} \\ \dot{\delta BGS} \end{bmatrix} = F_1 \begin{bmatrix} \delta K \\ \delta BKS \\ \delta BGS \end{bmatrix} + G_1 \begin{bmatrix} \delta NKS \\ \delta NGS \end{bmatrix} \tag{8}$$

where F_1 is

$$\begin{bmatrix} \overline{BKS}/COR(\overline{BKS}+\overline{BGS})-1/ALC & \overline{K}\times\overline{BGS}/COR(\overline{BKS}+\overline{BGS})^2 & -\overline{K}\times\overline{BKS}/COR(\overline{BKS}+\overline{BGS})^2 \\ -\overline{BKS}/COR(\overline{BKS}+\overline{BGS}) & -\overline{K}\times\overline{BGS}/COR(\overline{BKS}+\overline{BGS})^2 & \overline{K}\times\overline{BKS}/COR(\overline{BKS}+\overline{BGS})^2 \\ -\overline{BGS}/COR(\overline{BKS}+\overline{BGS}) & \overline{K}\times\overline{BGS}/COR(\overline{BKS}+\overline{BGS})^2 & -\overline{K}\times\overline{BKS}/COR(\overline{BKS}+\overline{BGS})^2 \end{bmatrix}$$

$G_1 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\overline{X}$ denotes the value of state X at the operating point. Let $\varphi = (\overline{K}/COR)/(\overline{BKS}+\overline{BGS})$. Note that $\varphi = NCAT\times\overline{FDS}$ where $\overline{FDS}$ is the fraction of demand satisfied at the operating point. Also, let $\pi = \overline{BKS}+\overline{BGS}$ and note that $\pi = NCAT\times\overline{P^*}$, where $\overline{P^*}$ is the desired production at the operating point. Then, we have

$$F_1 = \begin{bmatrix} \overline{BKS}/COR\times\pi - 1/ALC & \varphi\times\overline{BGS}/\pi & -\varphi\times\overline{BKS}/\pi \\ -\overline{BKS}/COR\times\pi & -\varphi\times\overline{BGS}/\pi & \varphi\times\overline{BKS}/\pi \\ -\overline{BGS}/COR\times\pi & \varphi\times\overline{BGS}/\pi & -\varphi\times\overline{BKS}/\pi \end{bmatrix}$$

For FDS=1, the entire model is linear, and for FDS<1, the model is highly nonlinear. Note the following observations from the analysis of the above models:

1. FDS = 1

a. The system is stable with time constants ALC (average lifetime of capital, and NCAT (normal capital acquisition time): In this regime, excess capacity permits all orders to be delivered within the normal capital acquisition time, and capital stock depreciates with time constant ALC.

b. K and BKS are controllable via NKS. BGS is uncontrollable but stable. In other words, K can be controlled by NKS, but as long as FDS=1, the goods sector gets all it desires, and thus is not controllable. BGS is also decoupled from K and BKS.

2. FDS < 1

The eigenvalues are 0, $-\varphi$, and $\overline{\text{BKS}}/\text{COR}\times\pi-1/\text{ALC}$.

a. The Jacobian matrix has a zero eigenvalue, which implies that higher order terms ignored by the linearization are important. Thus, no stability conditions can be inferred from the Jacobian matrix. Due to its highly nonlinear nature, it seems to be safer to keep the system away from this region whenever possible.

b. The linearized system is controllable. Therefore, the goal is to design a controller which stabilizes the system in this region, or better still, gets the system out of this region into the stable region where FDS=1.

An interesting characteristic of this model is that the equilibrium point, which lets us calculate the nominal trajectory for a given NGS, is exactly at the boundary of these two regions, making the system harder to control. Specifically, at the equilibrium,

$$FDS = 1 \tag{9.a}$$

$$K_n = \frac{ALC \times COR}{ALC-COR} \times NGS \tag{9.b}$$

$$BKS_n = \frac{NCAT \times COR}{ALC-COR} \times NGS \tag{9.c}$$

$$NKS_n = \frac{COR}{ALC-COR} \times NGS \tag{9.d}$$

$$BGS_n = NCAT \times NGS \tag{9.e}$$

where X_n denotes the nominal trajectory for X.

Thus, to sustain the nominal trajectory given NGS, NKS should be chosen as $\frac{COR}{ALC-COR} \times NGS + \delta NKS$ where δNKS is chosen in order to control the deviations from the nominal trajectory.

Controller Design:

The model, implemented in STELLA, was extended to support two controllers (one for each region), to perform pole placement using full state feedback. The controller determines the new orders of the capital sector NKS and is designed to calculate H_i at every instant of time such that the eigenvalues of $F_i+G_iH_i$ are as desired, for each region, i = 1, 2. The nominal trajectory is given by the equilibrium rate of capital sector orders given the current orders of the consumer sector. Then, the resulting input is:

$$NKS = \frac{COR}{ALC-COR} \times NGS + H_i \begin{bmatrix} \delta K \\ \delta BKS \\ \delta BGS \end{bmatrix} = NKS_n + \delta NKS_i \tag{10}$$

Note that as long as the desired poles are chosen to be nonzero and stable, the trajectories will be stable around the slowly varying nominal trajectories. In the control literature, the system is usually linearized around the equilibrium point and the poles are set accordingly. Here, we place the poles for all the system (except perhaps at a few number of points where controllability is lost). Although this is not backed up by rigorous results, it merits further research in view of the successful results shown below.

Figure 2 shows a policy structure diagram of the overall controller (equations for the controller are supplied in the Appendix). First, H_1 and H_2 are calculated on the basis of the current state of the system and the desired closed loop poles. The entries of H_i were found such that the characteristic polynomial of the closed loop matrix (determinant of $\lambda I-(F+GH_i)$) is the desired polynomial ((λ-desired pole 1)(λ-desired pole 2) etc.). Then, δNKS_1 and δNKS_2 are calculated as the weighted sum, by multiplication via H_1 and H_2 respectively, of the deviation from the nominal trajectory, which are determined by the current value of NGS and Equation (9). This stabilizes the dynamics of the error modes of the closed loop system and thus drives the states to the steady state values. Thus, the system is more robust since the error will go to zero and the response will not be very sensitive to perturbations in H_1 and H_2. Finally, δNKS_1 or δNKS_2 is chosen depending on the value of FDS, and NKS_n is added to calculate NKS.

Three poles in the region FDS < 1 were placed in the left half plane on the circumference of a circle centered at the origin. One pole was placed on the real axis with a complex conjugate pair with equal real and imaginary parts. The radius of the circle determines a trade-off between overshoot and settling time (the larger the radius, the more overshoot and faster settling). In the region FDS = 1, two poles were placed on the real axis in the left half plane, both at the same location. Their distance from the origin determines the settling time in this region.

Positive shocks to NGS from equilibrium put the system in the unstable, nonlinear region FDS < 1. The controller is designed to move the system into the stable region in an "optimal" fashion - quickly and with minimum overshoot. In the region FDS = 1, the system is expected to settle at the equilibrium in an overdamped way, thus providing the overall desired response. Negative jumps in NGS put the system in the region FDS = 1. The response will be overdamped in that case.

In intuitive terms, increasing the speed of adjustment in the region FDS < 1 may result in building the capital stock too high, and thus degrading performance by increasing the score. Decreasing the speed when FDS < 1 may cause the system to linger in this region with inadequate investment to escape, also degrading performance by raising the score. Increasing the speed in the region FDS = 1 may cause the system to reenter the region FDS < 1 due to the discrete time intervals used in the Strategem-2 game. This is not expected to cause any problems since capital will be bounced back up again, but may cause small amplitude (probably damped) oscillations. Decreasing the speed in this region may cause slow settling to the equilibrium (overconservative response), and thus increase the score.

Experimental Results:

Various speeds were tried, and simulation results for pole locations -0.38, $-0.27 \pm 0.27j$ for FDS < 1, and -1, -1 for FDS = 1 are illustrated in Figure 3. In the figure, desired capital DesiredK = (BKS+BGS)/NCAT. The capital sector input supports the intuition that a positive jump in the goods sector demand is followed by an amplified positive jump in the control input NKS such that the share of the capital sector demand in the desired production is increased enough to increase the capital and supply the desired production (bounced to the region FDS = 1). Later, the input drops close to zero for a while to compensate for the initial overdemand, and finally settles to its equilibrium value. Capital shows the desired response, while compensating for the jump in the backlog of the goods sector. FDS falls to 80%, but recovers in about 10 years. The score for the simulation is 15.[2,3]

To investigate robustness the speed inputs were perturbed by 10% in both directions independently. This resulted in negligible changes in the response, and the increase in the average score per

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[2]In the Strategem-2 game, all quantities are rounded to the nearest 10 units to simplify the decisionmaking task of the subjects. The simulation analyzed here permits the states to take continuous values. As a result, the score in the simulation can be less than the optimal score of 19 in the experiment.

[3]Compare the behavior in Figure 3 to Figure 4 which shows the cycles typically produced by subjects of the experiment.

period was less than 1 for all perturbations.

Another robustness test was performed by introducing uncertainty in choosing the appropriate controller around the equilibrium. Specifically, this boundary was chosen as the region: $0.95 \leq$ (desired production)/(production capacity) ≤ 1.05. Figure 5 shows the simulation result when the controller for FDS < 1 was chosen for this boundary. The response exhibits some underdamped fluctuations due to the complex conjugate pair, but they seem to be unimportant. Figure 6 shows the simulation result when the controller for FDS = 1 was chosen for this boundary. Since the controller for FDS = 1 was overdamped, when used in the boundary, it never bounces the system into the region FDS = 1. Thus, FDS never makes it to 1, and gets into a cycle where it slides down, jumps up (but below 1), slides back down, etc. Recall that there was an extra degree of freedom in the controller for FDS = 1, which was not used, since BGS is uncontrollable. Specifically, BGS was not included in the controller. To correct for the above deficiency, a multiple of BGS was added to the controller for FDS = 1. This has no effect in the region FDS = 1, since the capital sector is decoupled from the goods sector in this region. In the boundary, this could be used to correct the undesired effect of the uncertainty. Figure 7 illustrates the simulation result when 12% of BGS was added to the controller of the region FDS = 1. The response is greatly improved, and FDS settles at 1 in this case.

Intuitively, when faced with an unstable region and uncertainty in the states, a robust controller will err on the side of caution by moving the system rapidly to the stable zone even at the cost of possibly moving farther into the stable zone than necessary. The results for other simulations where different waveforms for the goods sector demand were chosen are summarized in Table 1.

The controller seems to be robust with respect to perturbations in the desired pole locations, which also correspond to perturbations in the feedback gains. The discrepancy due to the uncertainty in choosing the appropriate controller around equilibrium was corrected by using the additional degree of freedom in the region FDS = 1. Thus the results are not contingent on precise knowledge of the system parameters.

The controller designed in this paper could be modified to follow ramp inputs better, but is likely to give worse results in response to other inputs. More advanced control techniques, such as linear quadratic regulation, could be applied to this problem, but requires the use of packaged programs to solve the necessary equations.

Conclusions and Further Analysis

This paper presents a heuristic application of control theory to an economic model. Although we considered one case, the same approach can be applied to a wide class of problems which exhibit the same structure, that is, any system with a region in which resources are fully utilized and a region of slack. In general, the results show that a problem which is highly nonlinear can be controlled quite satisfactorily with the aid of simple mathematical analysis and basic control concepts. We have considered two key techniques for approaching such nonlinear problems:

1. Linearization around the nominal trajectory: This is a commonly used technique. The results of Vidyasagar (Vidyasagar 1978), although complicated, are particularly useful in finding the class of "slowly varying" trajectories that the system would be stable around. We verified this for step inputs by simulation. Ramp inputs and sinusoids seem to present some problems.

2. Analyzing piecewise-linear nonlinearities in linear regions separately: We considered a common example of this case, saturation. However, since the general applications of control theory require the system to operate in the unsaturated region, there is not much rigorous analysis to be found in the literature for the case when the operating point is required to be on the boundary between the two regions, as in many system dynamics problems. This perhaps merits a rigorous analysis.

Comparing the "Optimal" Policy with Actual Decision-Making Practice:

Many economists argue that dysfunctional oscillations such as the long wave cannot exist since economic agents with rational expectations would behave in an optimal fashion. The analysis here shows that there are in fact optimal strategies for investment which can avoid instability. But, it is very unlikely that the results presented in this paper would be achieved in practice through conventional decision making processes. This assertion is supported by the experimental results in Sterman 1987 and Sterman 1988a. The decision rule developed here requires information a manager in real life is unlikely or unable to have. It then processes that information in a highly sophisticated fashion. In particular, the optimal rule utilizes knowledge of the equilibrium structure of the system to compute targets for capital stock and new orders. The equilibrium structure of the full economy is not known to real firms. The rule also requires very different strategies depending on which regime one is operating in, requiring a firm to be able to decide whether there is excess or insufficient capacity compared to demand. But in reality, an individual firm is unable to tell whether customers' orders represent long run needs, transient stock adjustments, or self-ordering effects. The optimal rule is not fooled, as players of the Strategem-2 game frequently are, into ordering still more in response to the rise in demand induced by their own attempt to raise capacity. It is unlikely that actual firms use dramatically different decision rules in each regime, particularly since a firm cannot distinguish with certainty which regime the economy is operating in. Rather, as experiments with managers, economists, and students confirm, people tend to use a single decision process which is locally rational (but leads to poor performance in the full system).

To overcome the large gap between the requirements of optimal control and the reality of bounded rationality, we stress that in problems of this type, the trend of the inputs is much more important in practice than precise values. The tests of controller robustness also suggest that small variations due to modelling errors, etc. still produce very satisfactory results. The main result of the analysis is the critical role of nonlinearity in determining the best strategy. Performance can be dramatically improved by determining which regime one is operating in and reacting accordingly, even if the decision rules for each regime are only approximately correct. As in any policy analysis, implementation will depend on the modeller's ability to explicate the rationale for the policy. The use of control concepts reinforces rather than replaces the need for managerially oriented justification of the proposed policies.

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APPENDIX

Equations for the Controller

FDS <1:

Given desired polynomial $\lambda^3+a\lambda^2+b\lambda+c$, the feedback vector is:

$H_1 = [\ h_1\ h_2\ h_3]$ where

$h_1 = (\pi/(\varphi\times\overline{BGS}))(r\times\varphi - r^2 + a\times r - b - COR\times c/(\varphi\times\overline{BGS}(\pi - COR)))$

where $r = -\overline{BKS}/(COR\times\pi) + 1/ALC$

$h_2 = \varphi + r - a$

$h_3 = COR\times c/(\varphi\times\overline{BGS}\times(\pi - COR)) - (\overline{BKS}/\overline{BGS})(\varphi + r - a)$

FDS $= 1$:

Only two poles can be placed. Thus, given a desired second order polynomial $\lambda^2+a\lambda+b$, the feedback vector is:

$H_2 = [h_1\ h_2\ h_3]$ where

$h_1 = a\times NCAT/ALC - NCAT/ALC^2 - b\times NCAT$

$h_2 = a - 1/NCAT - 1/ALC$

The value of h_3 does not effect the characteristic polynomial. Recall that we used this degree of freedom to improve robustness.

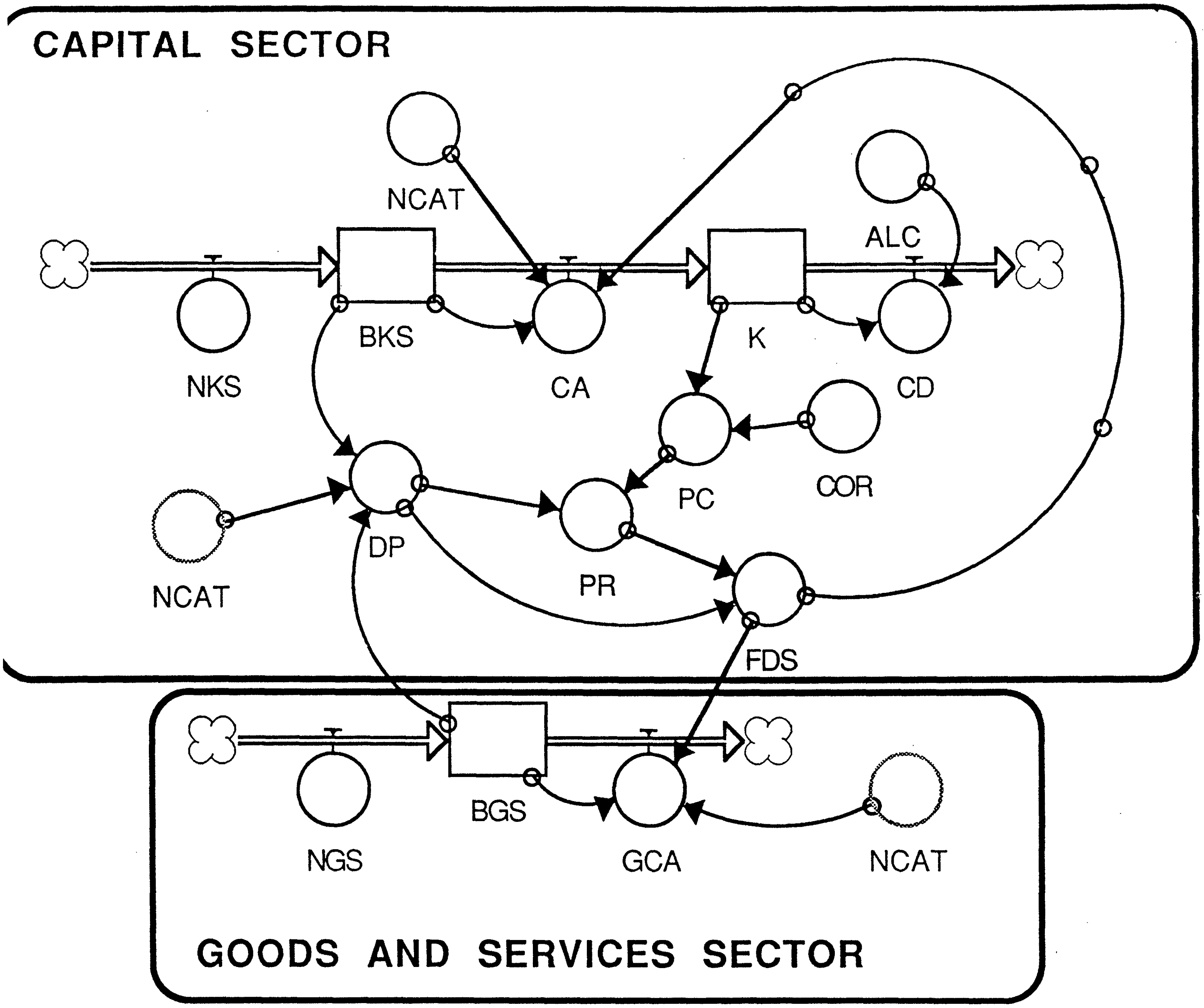


Figure 1.a Simulation Model

Equations of the Long Wave model: version to simulate the STRATEGEM-2 game. New orders for capital placed by the consumer goods and services sector (NGS) are exogenous. In the experiment, the new orders of the capital sector (NKS) are determined by the player. In the analysis presented here, NKS is determined by the controller.

Capital Sector:

FDS = PR/DP	Fraction of Demand Satisfied (dimensionless)
DP = (BKS+BGS)/NCAT	Desired Production (units/year)
NCAT = 2	Normal Capital Acquisition Time (years)
PR = MIN(DP,PC)	Production Rate (units/year)
PC = K/COR	Production Capacity (units/year)
COR = 2	Capital Output Ratio (years)
K = K + dt * (CA - CD)	Capital Stock (units)
CD = K/ALC	Capital Discard Rate (units/year)
ALC = 20	Average Life of Capital (years)
CA = FDS*BKS/NCAT	Capital Acquisition Rate (units/year)
BKS = BKS + dt * (NKS - CA)	Backlog of orders of the capital sector (units)
NKS = Determined by subject	New Orders of the Capital Sector (units/year)

Goods and Services Sector:

BGS = BGS + dt * (NGS - GCA)	Backlog of orders of the goods sector (units)
GCA = FDS*BGS/NCAT	Goods Sector: Capital Acquisition Rate (units/year)
NGS = Exogenous	New Orders of the Goods Sector (units/year)

dt = 2	Simulation time step (years)

Figure 1.b Equations of the Long Wave Model

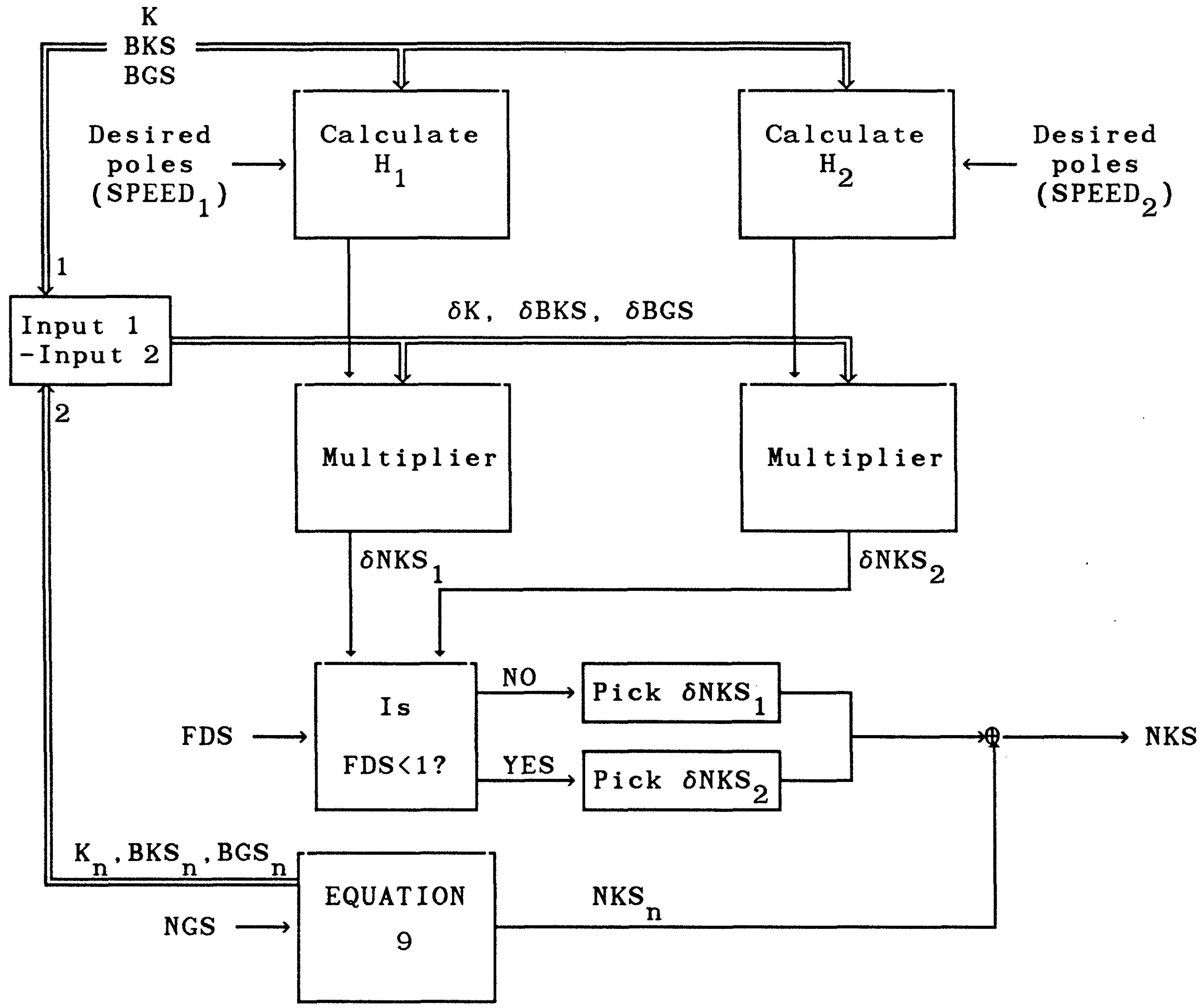


Figure 2 Block diagram of controller.

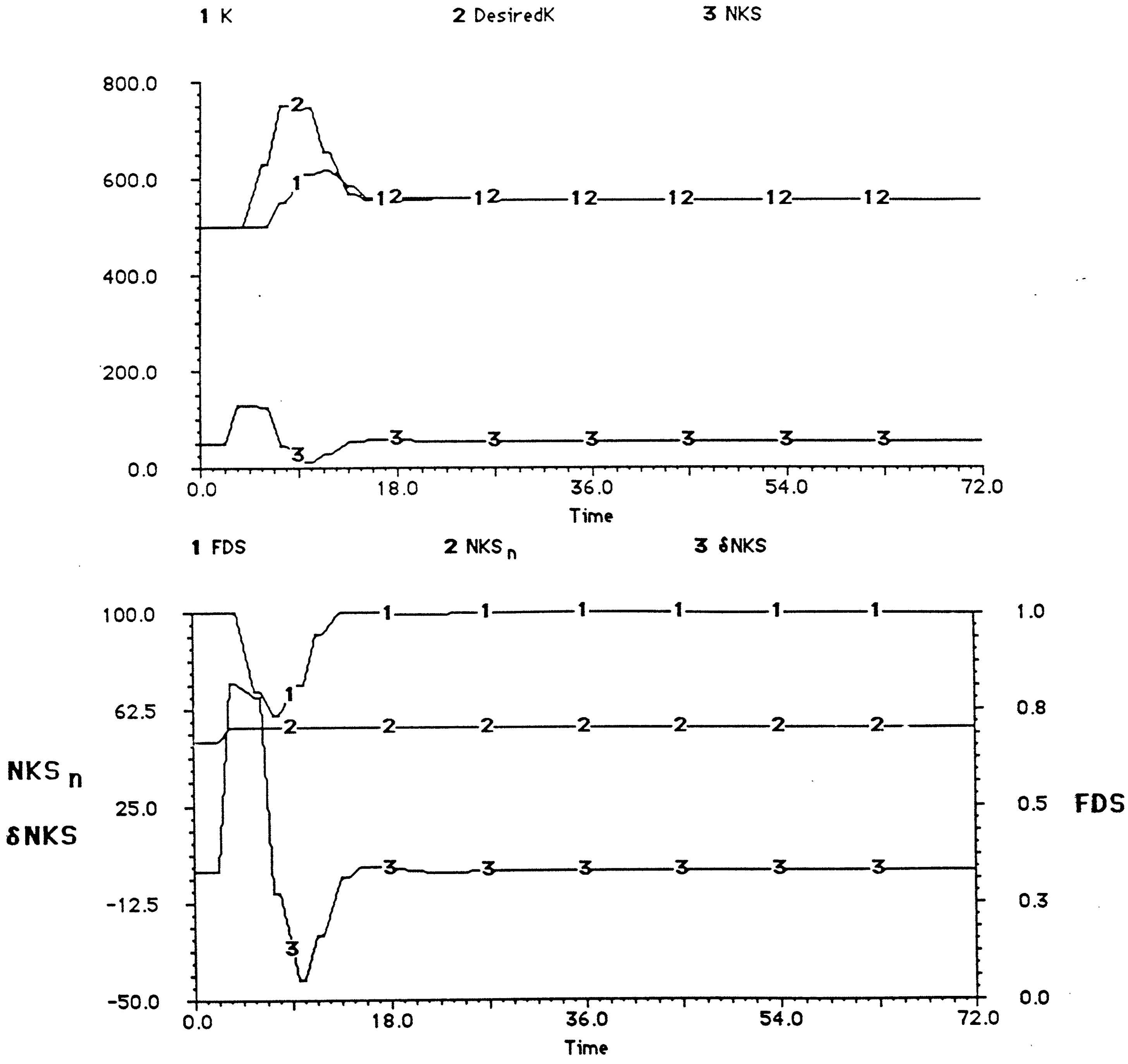


Figure 3 Simulation of the basic run: Response to a 10% step increase in demand. Score = 15; K, DesiredK, NKS: units; FDS: %.

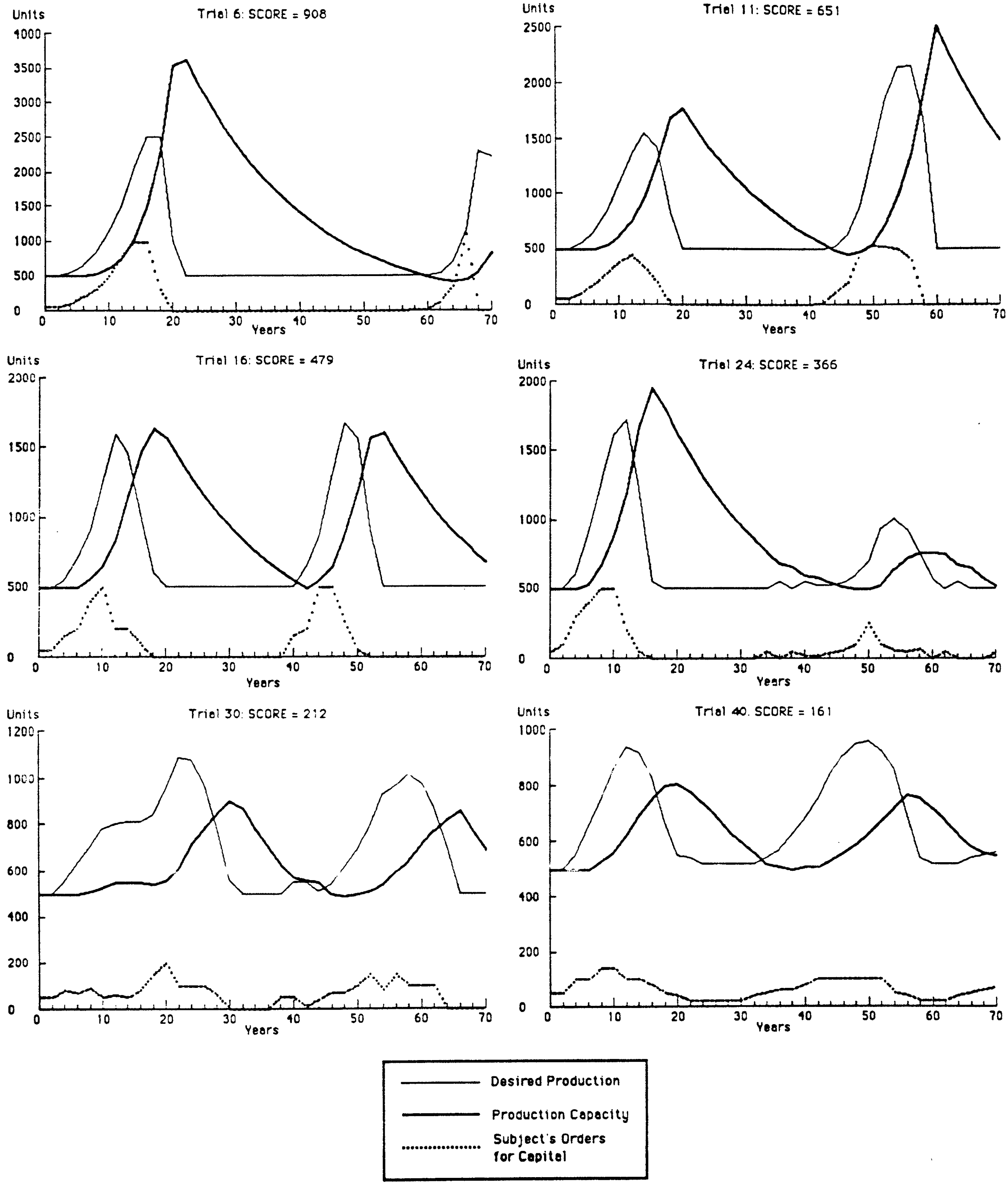
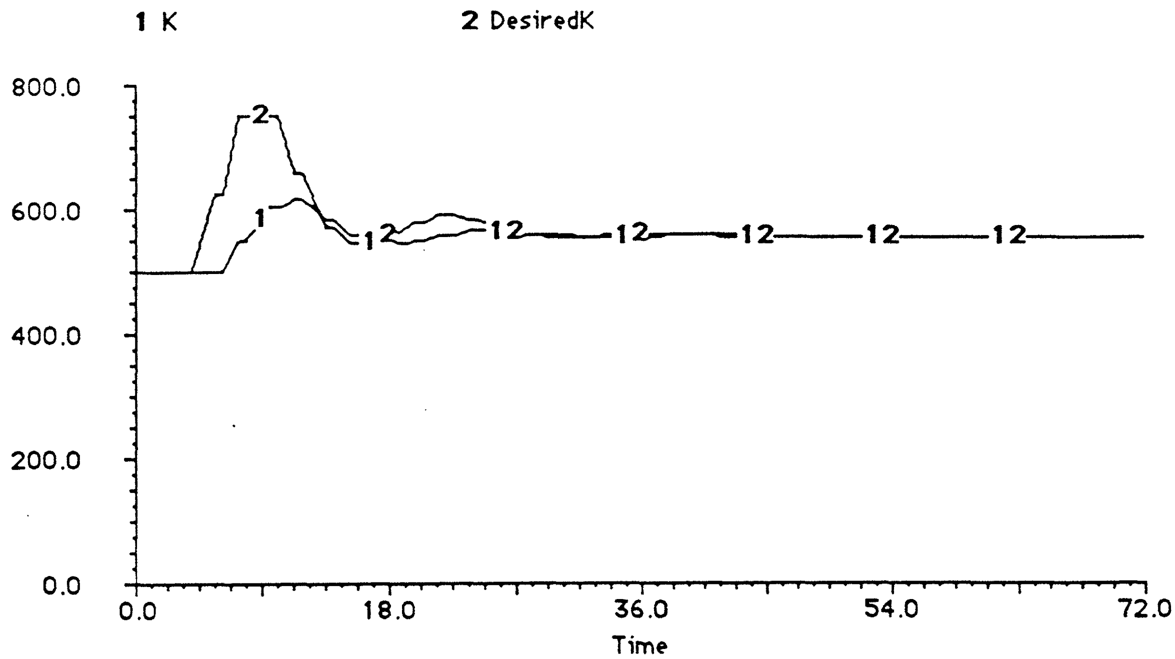


Figure 4 Typical experimental results. Note the large amplitude and long period of the cycles generated by the subjects. N.B.: vertical scales differ.



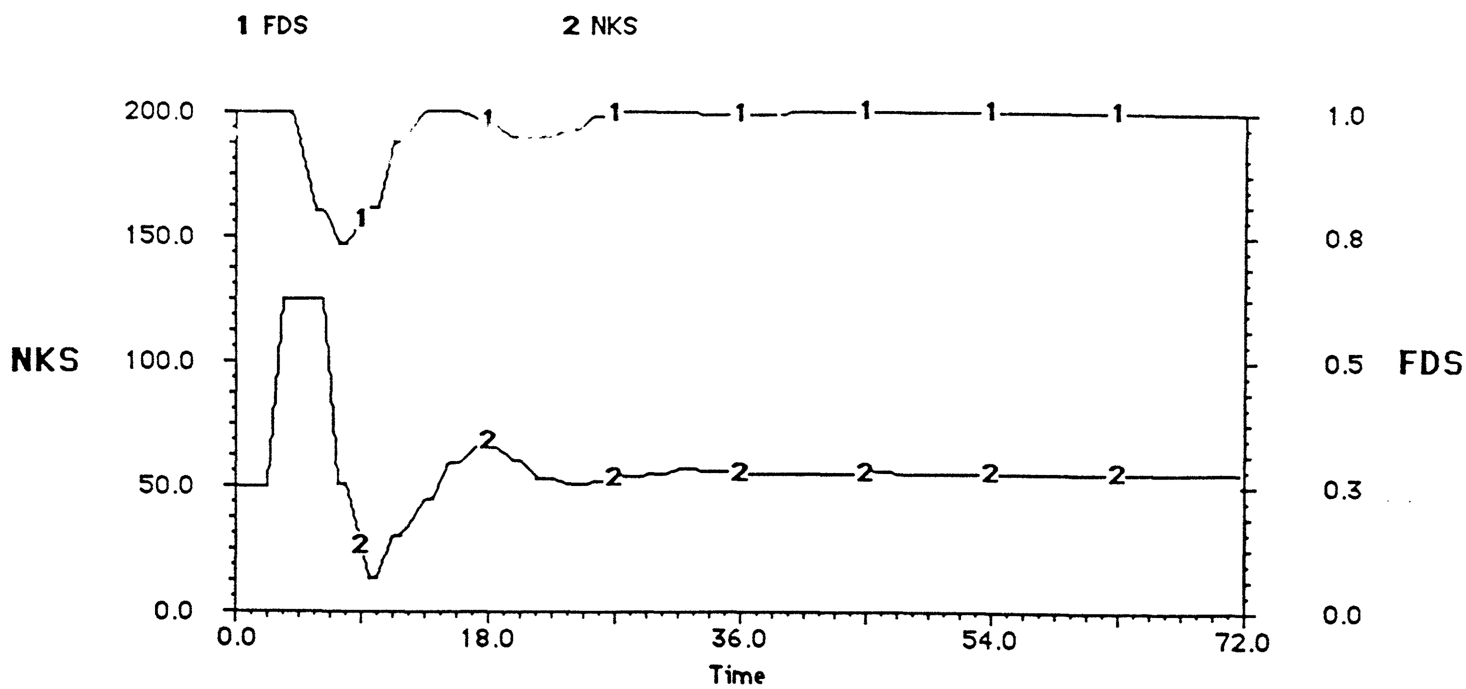


Figure 5 Controller for FDS<1 used in the boundary whenever .95 ≤ DP/PC ≤ 1.05. Score = 18; K, DesiredK, NKS: units; FDS: %.

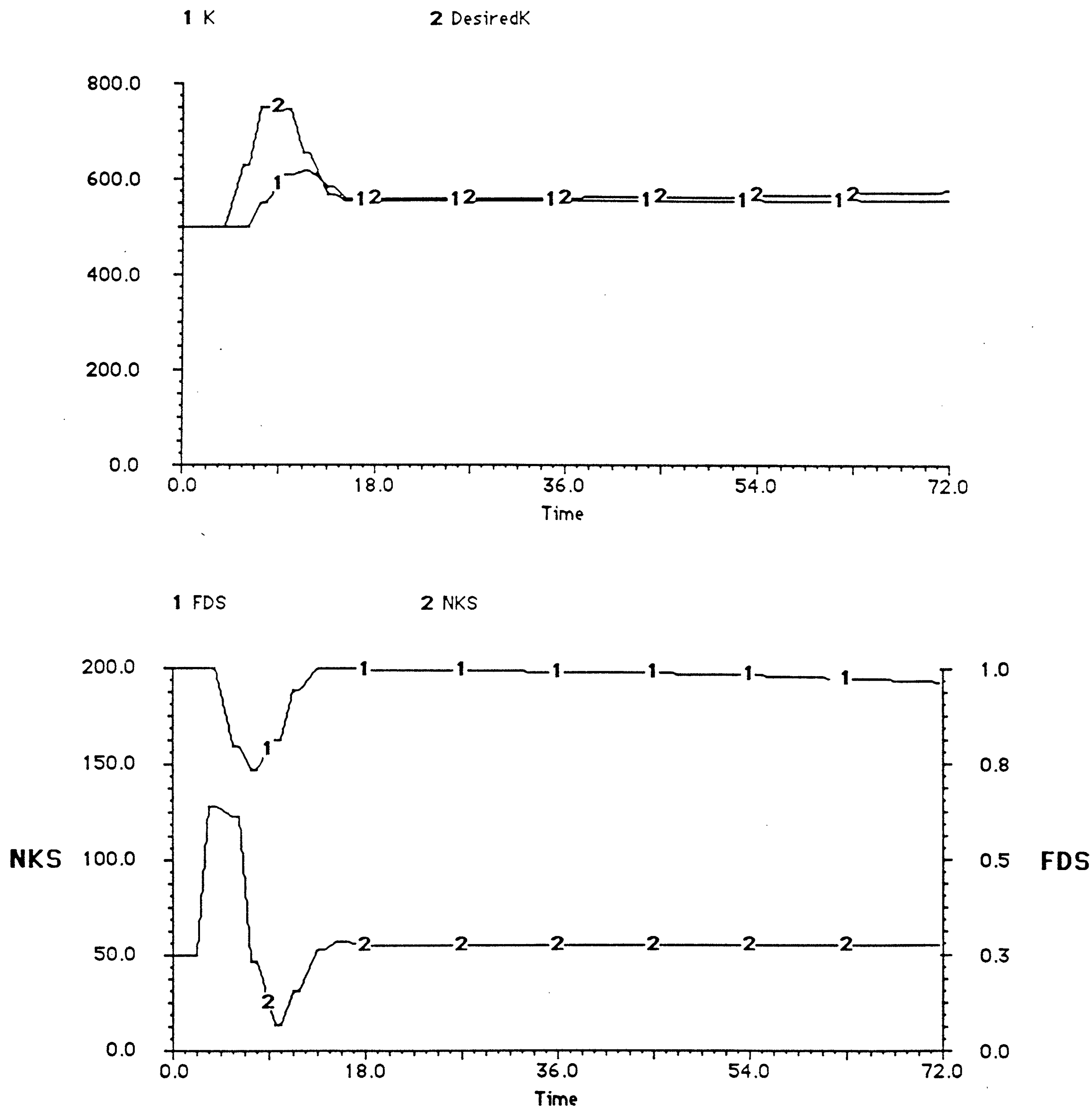
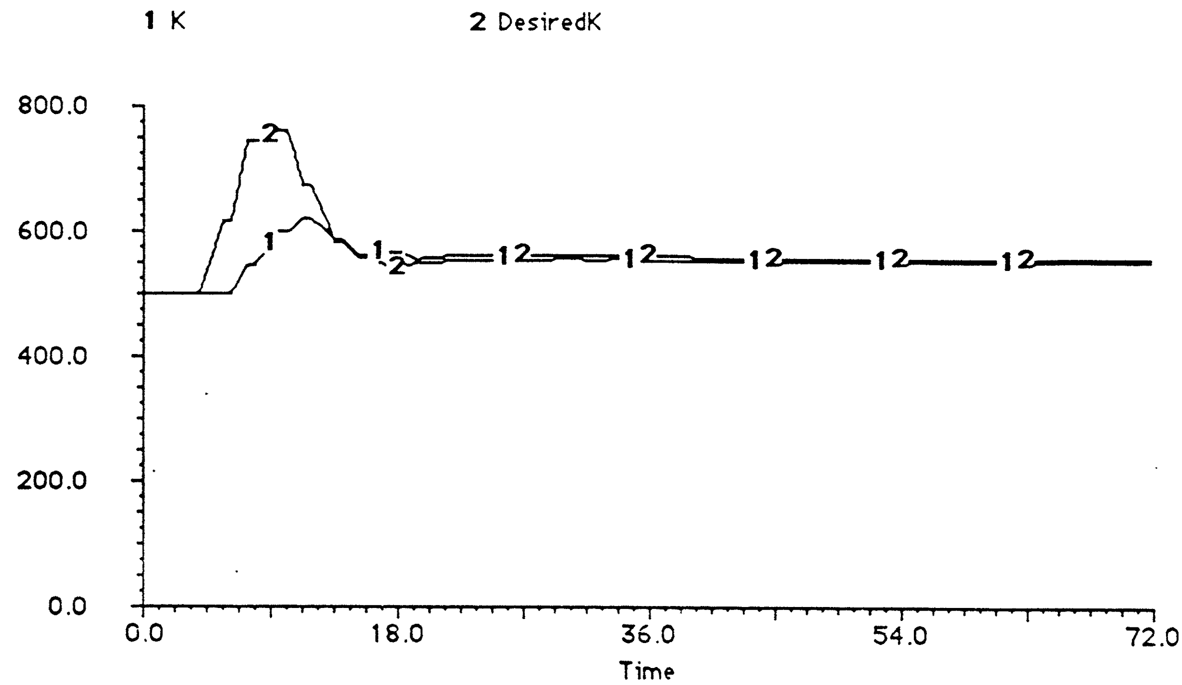


Figure 6 Controller for FDS=1 used in the boundary whenever $.95 \leq DP/PC \leq 1.05$. Score = 21; K, DesiredK, NKS: units; FDS: %.



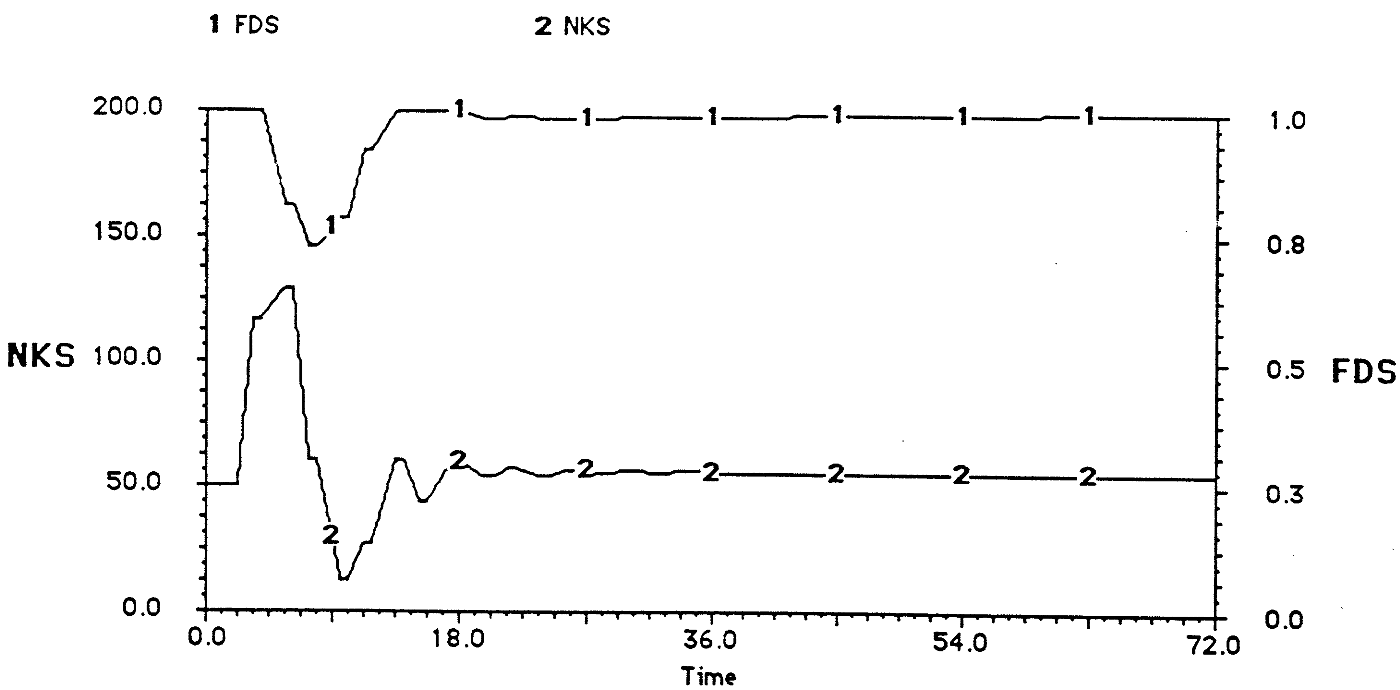


Figure 7 Improvement of controller for FDS=1 used in the boundary. Score = 19; K, DesiredK, NKS: units; FDS: %.

Table 1 Summary of simulations with various waveforms for NKS.

NGS	K	FDS	Avg score per period
Ramp $225+2.5(t-2)$	Ramp with a lag No equilibrium	Above 90%	Settles around 100
Sinusoidal $225+50\sin(\frac{2\pi t}{72})$	Distorted sinusoidal	Periodic between 100% and 70%	Settles around 125
Gaussian Noise $225+20\times$NORMAL	Noisy but stable	Fluctuates between 100% and 60%	Below 200